Remote sensing satellite virtual constellation optimizing with target recognition probability

Alexsandr M. Kondratov, Oleg V. Maslenko

Abstract**—**The improved method of satellite systems selection for the electro-optical observation of the specified objects and regions of the Earth is described. The method provides the probability of targets correct detection (recognition). Other indicators of system selected efficiency, which reflect the controlled area, the timeliness of space imagery and the cost should not be less than a predetermined threshold.

Keywords**—**Earth observation, remote sensing satellite system, virtual constellation, target recognition probability.

I. INTRODUCTION

Currently, the market of information services in the field of remote sensing of the Earth is developing rapidly. Applications for satellite imagery of the necessary regions of the Earth are realized by the electro-optic imaging satellite systems. Satellite images are distributed by dealers that provide their services in Internet. They offer both new-ordered and archive images for anywhere Earth territory. When ordering the satellite images (both new and archive), a great number of parameters that affect the efficiency of the mission accomplishment should be taken into account.

The main elements of any satellite system for electro-optical surveillance are spacecraft and imaging equipment mounted on them. The satellite system selection depends on the geographical, geometrical and physical characteristics of areas and objects of interest; imaging conditions (time of year, time of day, state of the atmosphere, cloudiness level); technical specifications of the imaging equipment; spacecraft position in space and time in relation to the areas of interest and data reception points; state affiliation of the satellite system owner; information requirements (accuracy, reliability, timeliness of acquisition, cost). Therefore, special algorithms are required for optimal solving the problem of a suitable satellite system selection.

The most common way to forecast the efficiency of the remote sensing task solution is the service simulation of satellite system application [1]. During the simulation the swath of Earth’s surface and their positioning on the area of interest are computed. The time intervals when imaging is possible are calculated. The orbits by which the spacecraft passes over the area of interest at a specified time are selected. The illuminance conditions of the area of interest, the viewing angle inclination, imaging repeat cycle, the cost of a satellite imagery scene and other parameters are determined [2, 3]. Thereafter, the problem of the most suitable satellite systems selection is solved. By comparison, the following indicators of the efficiency of satellite observation systems are analyzed: the time of data obtaining and the information renewal rate; productivity of a satellite system that is specified as the total area imaged per day; spatial resolution of the satellite image; positional accuracy of the captured images in geodetical coordinate system, the geometric distortions of image, etc. [4]. Most of the available techniques for satellite observation planning rely on sophisticated and accurate models of the spacecraft orbital motion, however, as a rule, without consideration the probabilistic characteristics of the target detection and recognition [5].

II. METHOD

To determine the expected time of satellite imaging, it is necessary to set the geographical coordinates of the area of interest and to define the moments when the path of viewing axis crossing the area boundaries [6, 7]. Initial data are the Kepler elements of solar synchronous orbits. The inclination of the viewing axis for the roll η and the current moment of the spacecraft flying time t_j, $j = 0, 1, 2, \ldots$ are considered to be known. Required values are the geographic coordinates of the observation point: geographic latitude $\varphi_j = \varphi(t_j)$ and longitude $\lambda_j = \lambda(t_j)$ at specific time interval t_j within the boundaries of the orbit selected. At first, the coordinates of the viewing point are calculated in accordance with a spacecraft coordinate system. Then, the coordinate system is converted to the other one, in which the geographic coordinates of the viewing point are calculated. Further, the path of the viewing axis is designed as a set of the viewing points with allowance for the flying time. Current coordinates of the viewing points in a geocentric spherical coordinate system at a time point t_j [6, 8] are following:

$$\varphi_j = \arcsin\ (\sin u_j \sin i \cos\psi_\eta + \cos i \sin \psi_\eta) \tag{1}$$

$$\lambda_j = \pm\arccos\left(\frac{(\mathrm{a}_{11}\cos\psi_\eta + a_{31}\sin\psi_\eta)\cos\theta_{ç} + (\mathrm{a}_{12}\cos\psi_\eta + a_{32}\sin\psi_\eta)\sin\theta_{ç}}{\sqrt{(\mathrm{a}_{11}\cos\psi_\eta + a_{31}\sin\psi_\eta)^2 + (\mathrm{a}_{12}\cos\psi_\eta + a_{32}\sin\psi_\eta)^2}}\right) \tag{2}$$

where ψ_η is the geocentric angle between the radius-vector of a spacecraft and the radius-vector of a viewing point, a_{11}, a_{12}, a_{31}, a_{32} are the elements of transition matrix from the inertial to the Greenwich geocentric coordinate system.

The calculation of a latitude argument at a time point t_j is carried out according to the formula:

$$u(t_j) = \sqrt{\frac{\mu_0}{a^3}}\Delta t_j \tag{3}$$

where $\Delta t_j = t_j - t_\Omega$ is the time interval since the moment the spacecraft was located in the ascending orbit.

An optical axis path makes it possible to set the time when satellite survey begins as a time point when the path coordinates match the geographical coordinates of the area of interest at a certain orbit.

Inasmuch as the area of interest is defined as a trapezoid mostly, the start of satellite survey can be set as [9]:

$$t_m^{\mathrm{sa}}(n) = \begin{cases} t_m^S(n), \text{ if } (\varphi_j = \Phi_m^S) \wedge (\Lambda_m^W \le \lambda_j \le \Lambda_m^E) \wedge (S \to N) = 1 \\ t_m^W(n), \text{ if } (\lambda_j = \Lambda_m^W) \wedge (\Phi_m^S \le \varphi_j \le \Phi_m^N) \wedge (S \to N) = 1 \\ t_m^N(n), \text{ if } (\varphi_j = \Phi_m^N) \wedge (\Lambda_m^W \le \lambda_j \le \Lambda_m^E) \wedge (N \to S) = 1 \\ t_m^E(n), \text{ if } (\lambda_j = \Lambda_m^E) \wedge (\Phi_m^S \le \varphi_j \le \Phi_m^N) \wedge (N \to S) = 1 \end{cases} \tag{4}$$

where Φ_m^S and Φ_m^N are the southern and northern latitudes of the spherical trapezium sides of the *m*-th region, Λ_m^W and Λ_m^E are the western and eastern longitude of the spherical trapezium sides of the *m*-th region, $t_m^S(n)$, $t_m^N(n)$ are the time points when the sighting axis path crosses the southern and northern boundaries of the *m*-th region at the *n*-th orbit of the spacecraft, $t_m^W(n)$, $t_m^E(n)$ are the time points when the sighting axis path crosses the western and eastern boundaries of the *m*-th region at the *n*-th orbit of the spacecraft, $S \to N$ is the spacecraft orbital motion from south to north, and $N \to S$ from north to south.

The region area *S* defined as the spherical trapezoid can be calculated [9] by formula:

$$S(\Phi, \Lambda) = R^2\,(\Lambda_m^E - \Lambda_m^W)\cdot(\sin\Phi_m^N - \sin\Phi_m^S) \tag{5}$$

where *R* is the Earth's radius.

The instantaneous projection area of the imaging coverage in case of the viewing axis inclination from the nadir in relation to the flat Earth's surface can be calculated using a formula:

$$S(\eta) = H^2 \operatorname{tg}\beta\,[tg(\alpha+\eta)+\operatorname{tg}(\alpha-\eta)]\cdot[\sec(\alpha+\eta)+\sec(\alpha-\eta)] \tag{6}$$

The presence or absence of visibility conditions of the μ-th spacecraft for the *m*-th region during the satellite observation can be determined [6] using a logic function

$$\Phi_m^F(n_\mu)=\begin{cases}1,\text{if } (K_m^S(\mu)\geq \overline{K}_m^S)\wedge(K_m^T(\mu)\geq \overline{K}_m^T)\wedge(\beta_m^c(\mu)\geq \overline{\beta}_m^c)\wedge(Q_m^\xi(\mu)\leq \overline{Q}_m^\xi)=1\\ 0,\text{if } (K_m^S(\mu)\geq \overline{K}_m^S)\wedge(K_m^T(\mu)\geq \overline{K}_m^T)\wedge(\beta_m^c(\mu)\geq \overline{\beta}_m^c)\wedge(Q_m^\xi(\eta)\leq \overline{Q}_m^\xi)=0\end{cases} \tag{7}$$

where $K_m^S(\mu)$ is the spatial coefficient of the *m*-th region coverage by the swath of the μ-th spacecraft, which should not be less than the pre-defined valid value K_m^S. It can be found as a ratio of the expected area of the imaging of the *m*-th region by μ-th the spacecraft to the total area of the region:

$$K_m^S(\mu) = S_m(\mu)/S_m\ ,\quad K_m^S(\mu)\to\max \tag{8}$$

Secondly, $K_m^T(\mu)$ is the time coefficient of the *m*-th region coverage by the swath of the μ-th spacecraft, which should not be less than the pre-defined valid value $\overline{K}_m^T$.

The conditions of the temporal cover of the *m*-th region by the swath of the μ-th spacecraft can be submitted [6] as

$$[T_m(\mu)\in \overline{T}_m]\wedge[T_m(\mu)\geq \overline{T}_m]=1 \tag{9}$$

where $T_m(\mu)=t_m^{end}(\mu)-t_m^{start}(\mu)$ is the duration of an expected imaging interval, $\overline{T}_m=\bar{t}_m^{end}-\bar{t}_m^{start}$ is the duration of a specified imaging interval, $t_m^{end}(\mu)$, $\bar{t}_m^{end}$ are the expected and specified imaging termination time, $t_m^{start}(\mu)$, $\bar{t}_ò^{start}$ are the expected and specified imaging start time.

The temporal coefficient of coverage can be found as the ratio of the expected duration of the *m*-th spacecraft over the μ-th area $T_m(\mu)$ to the specified time of visibility of the area:

$$K_m^T(\mu) = T_m(\mu)/\overline{T}_m\ ,\quad K_m^T(\mu)\to\max \tag{10}$$

Thirdly, $\beta_m(\mu)$ is the current of the Sun's elevation angle for the satellite survey of the *m*-th region by the μ-th spacecraft, which needs to meet the requirement $\beta_m(\mu)\geq\overline{\beta}_m$, where $\overline{\beta}_m$ are the permissible minimum angle of the Sun's elevation [6]:

$$\beta_m(\mu)=\begin{cases}\dfrac{(t_{local}-t_{sunset})\,\beta_{\max}}{12-t_{sunset}} & \text{if } t_{local}<12^h\\[2ex] \dfrac{(24-t_{local}-t_{sunset})\,\beta_{\max}}{12-t_{sunset}} & \text{if } t_{local}\geq 12^h\end{cases} \tag{11}$$

where t_{local} is the current local time, t_{sunset} is local time of sunset; $\beta_{\max}$ is the maximum angle of the Sun's elevation within the area of interest.

In the fourth place, $Q_m^\xi(\mu)$ is the forecasted cloud-covered area of the *m*-th region for the μ-th spacecraft, taking into account the coefficient of transparency of the atmosphere ξ, which should be no less than the valid value $\overline{Q}_m^\xi$. Information about clouds over certain areas of the Earth can be obtained from the world meteorological services or other relevant institutions.

The expected cost of the image of the observed part of the *m*-th region, acquired by the μ-th spacecraft $C_m(\mu)$, can be estimated using the following data: the area of the *m*-th region observed by the μ-th spacecraft $S_m(\mu)$; commercial offers of the Earth observing systems operators concerning the minimum scene size in order $S_{\min}$; cost of an image C_1 per 1 sq. km; threshold area S_n, which exceeds the operator's ability to reduce the cost C_1; the cost of image acquisition C_2 per 1 sq. km with a discount ($C_2 \le C_1$).

Then the total image cost can be calculated by the following formula:

$$C_m(\mu) = \begin{cases} C_1 S_{\min} \text{ if } S_m(\mu) < S_{\min} \\ C_1 S_m(\mu) \text{ if } S_n > S_m(\mu) \ge S_{\min} \\ C_2 S_m(\mu) \text{ if } S_m(\mu) \ge S_n \end{cases} \tag{12}$$

Then the normalized cost factor of the satellite image should be used in the form

$$K_m(\mu) = \begin{cases} C_m(\mu) / C_m^{\max} \text{ if } C_m(\mu) < C_m^{\max} \\ 1 \text{ if } C_m(\mu) \ge C_m^{\max} \end{cases} \tag{13}$$

where $C_m^{\max}$ is the maximum cost of the image of the *m*-th area acquired by available spacecraft.

The coefficient (13) can take a range of values (14)

$$0 \le K_m(\mu) \le 1 \tag{14}$$

At the same time, the following term should be compiled with

$$K_m(\mu) \to \min \tag{15}$$

and the selection of suitable satellite system should be carried in accordance with smallest values of the coefficient (15) from the ordered set

$$K_m(1) < K_m(2) < \ldots < K_m(\mu) \quad , \quad \mu = \overline{1, \mathrm{M}} \tag{16}$$

The probability of correct detection (recognition) of the target $P(x, \theta)$ can be described by a generalized relation

$$P(x,\theta) \cong 1 - \prod_i \varepsilon(x_i, \theta_i) \tag{17}$$

where x is the input vector of optical signal, θ is the set of parameters, $\varepsilon(x, \theta)$ is the probability of error. The probability of error can be written as [10]

$$\varepsilon(x,\theta) \cong 1 - \Phi\left(\frac{\Delta x \sqrt{n}}{2\sigma}\right) \tag{18}$$

where $\Phi(x) = \frac{2}{\sqrt{2\pi}} \int_0^x e^{-\frac{u^2}{2}} du$ is the probability integral [11], Δx is the difference between target and background optical signals, σ is the standard deviation of the optical signals, n is the number of resolution elements within the target image.

Equation (17) describes the process of object detection. In passing to recognition, the dependence $\varepsilon(x, \theta)$ becomes more complicated. This fact can be taken into account using the Johnson criteria [12] or any other model that describes image recognition to the required information level.

III. RESULT

In this research, the model of satellite systems selection for the electro-optical imaging has been applied. For this purpose, the satellite survey of a specified area has been simulated. The area of interest was specified as shown in Fig. 1.

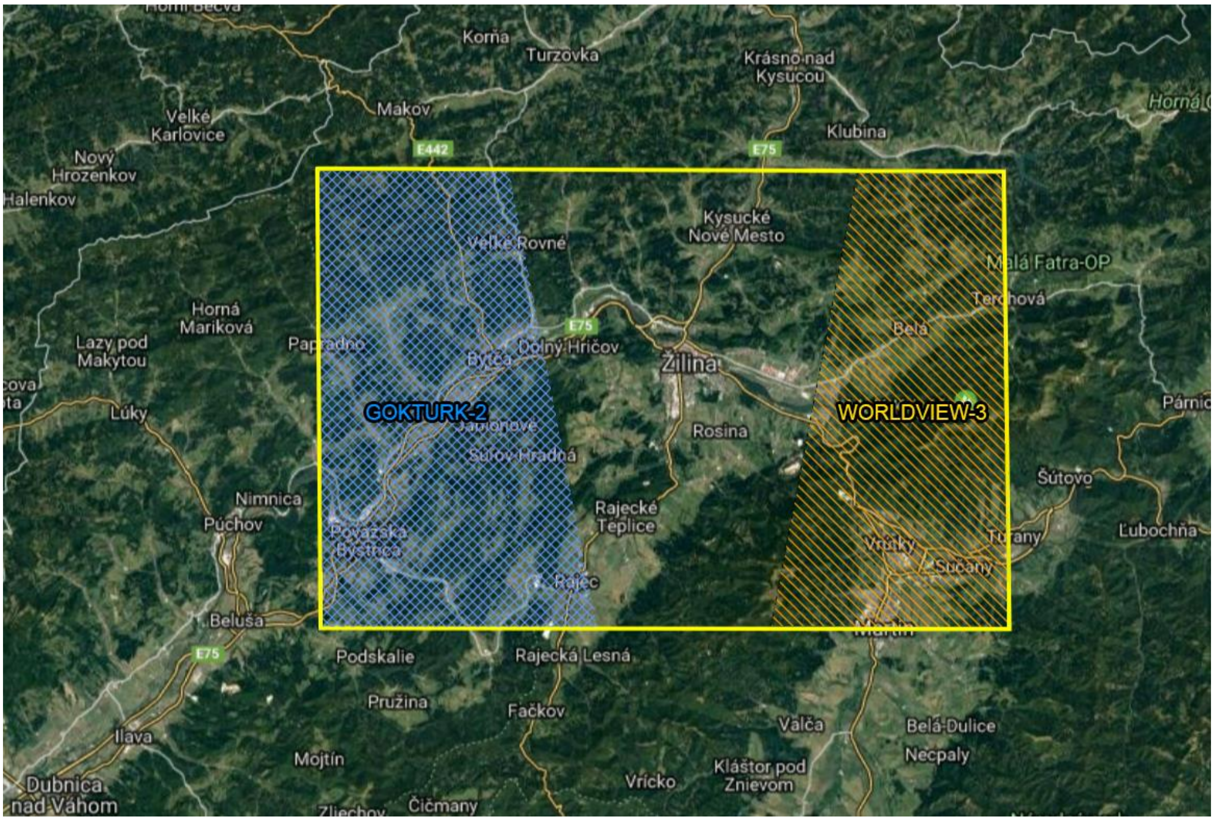

Fig. 1. The area of interest for satellite system selection

Also the some satellite system's swaths are plotted within the area of interest. The simulation's results are presented in the Table I.

TABLE I
THE RESULTS OF THE SATELLITE SYSTEMS EVALUATION

Time frame: form 6:00 a.m., 3 January 2019 till 6:00 a.m., 13 January, 2019			Time required to cover the region completely by one spacecraft swath	
Satellite system	Detection probability	Recognition probability	Hours	Days
WorldVie-1	0,9978	0,9912	122,4	5,1
Geoeye 1	0,9985	0,9941	123,912	5,163
Worldview-2	0,9981	0,9925	123,888	5,162
SPOT 6	0,9651	0,8681	3,24	0,135
GokTurk 2	0,9461	0,8017	74,208	3,092
KompSat 2	0,9912	0,9653	122,88	5,12
KompSat 3	0,9957	0,9828	77,592	3,233
Pleiades 1A	0,9978	0,9912	52,296	2,179
WorldView-3	0,9991	0,9966	148,344	6,181
KazEOSat 1	0,9912	0,9653	51,816	2,159
KazEOSat 2	0,6874	0,2244	27	1,125
Pleiades 1B	0,9978	0,9912	76,176	3,174
SPOT 7	0,9651	0,8681	27,144	1,131
NigeriaSat 2	0,9461	0,8017	98,832	4,118
Cartosat 2A	0,9963	0,9852	26,928	1,122
Cartosat 2B	0,9963	0,9852	74,904	3,121
DubaiSat 2	0,9912	0,9653	51,072	2,128
Deimos 2	0,9912	0,9653	123,192	5,133
WorldView-4	0,9991	0,9966	76,32	3,18
Sentinel 2A	0,4118	0,0291	28,272	1,178
Sentinel 2B	0,4118	0,0291	3,936	0,164

As it follows from the analysis of Table 1 data, the optimal satellite system for target detection is SPOT 6, and for target recognition is Cartosat 2A. Other satellite systems either do not meet the probability of detection (recognition), or consume more time to capture the entire area of interest.

IV. CONCLUSIONS

The improved method of satellite systems selection for the electro-optical observation of the specified targets and regions of the Earth is presented. The method provides the selection of electro-optical observation satellite systems with allowance for area of interest characteristics, conditions and timeliness of satellite survey, and also the cost of satellite imagery. The fundamental advantage of the method is the providing the probability required for targets correct recognition by the satellite imagery.

REFERENCES

[1] I.A. Glazkova, V.V. Malyshev, and V.V. Darnopykh, “Estimation of perspective micro-satellite Earth observation system efficiency on the base of imitative modeling (in Russian)”, *Computer Science and Control*, vol. 16, no. 6, pp. 125-134, June 2009.

[2] *STK User’s Guide*. Exton, PA: Analytical Graphics, Inc., 2004, 536 p.

[3] V.M. Vishnyakov, “Optimization of orbital constellation parameters of the satellite system for emergency monitoring (in Russian)”, *Current Problems in Remote Sensing of the Earth from Space*, vol. 1, no. 2, pp. 222-237, June 2005.

[4] O.D. Fedorovskyi, M.V. Artiushenko, and Z.V. Kozlov, “Parametric synthesis of space systems for remote sensing of the Earth on the basis of the genetic method (in Russian)”, *Space Science & Technology*, vol. 10, no. 1, pp. 54-60, March 2004.

[5] V. Malyshev, and V. Bobronnikov, “Mission planning for remote sensing satellite constellation”, in: *Mission Design & Implementation of Satellite Constellations*, ed. by Jozef C. van der Ha, Dordrecht: Kluwer Academic Publishers, 1998, pp. 431-437.

[6] P.V. Friz, *The Spacecraft Orbital Movement Fundamentals* (in Ukrainian), Zhytomyr: Korolev Zhytomyr Military Institute, 2012, 348 p.

[7] *Spacecraft Flight Theory Fundamentals* (in Russian), ed. by G.S. Narimanov, Moscow: Machine Building, 1972, 608 p.

[8] B.S. Skrebushevsky, *Spacecraft Orbits Formation* (in Russian), Moscow: Machine Building, 1990, 256 p.

[9] P.V. Friz, “Improved mathematical apparatus for determining the observed area of the landed earth region in space monitoring problems” (in Ukrainian), *The Journal of Zhytomyr State Technological University. Series: Engineering*, vol. 1, no. 2, pp. 126-134, Feb. 2017.

[10] S.A. Stankevich, “Estimating the linear resolution of digital aerospace images” (in Russian), *Space Science & Technology*, vol. 8, no. 2-3, pp. 103-106, May 2002.

[11] T.T. Soong, *Fundamentals of Probability and Statistics for Engineers*, Hoboken, NJ: Wiley, 2005, 498 p.

[12] T.A. Sjaardema, C.S. Smith, and G.C. Birch, *History and Evolution of the Johnson Criteria*, Oak Ridge, TN: Sandia National Laboratories, 2015, 40 p.