Improving the Accuracy of GPS Tracking Multiple Drones based on MCMC Particle Filter

Negm Eldin Mohamed Shawky

Abstract— GPS information received from multi-unmanned aerial vehicles (UAVs), also called drones via ground control station will be processed for detecting and tracking estimate target position. Tracking drones based on GPS has an issue of missed received information at some instant time or received information with an error that lead to lost tracking .Improving Position accuracy is the required measure of a system's capability to provide quality of location estimates, which in turn depends mainly on the type of selected algorithm. Markov chain Monte Carlo based particle filter (MCMC-PF) can be used to overcome the issue of loosing received information with keeping tracks and provide continuous tracking with higher accuracy. This is suitable for real time applications with moving multiple drones. Simulation results demonstrate the effectiveness and better performance when compared to conventional algorithm KF.

Keywords— data association, multi-target tracking, particle filter, Kalman filter, Markov chain Monte Carlo, GPS receiver, drones

I. Introduction

Unmanned Aerial Vehicles (UAV) or drone is a small aircraft without a pilot. This is now the most widely used in the military to provide a mission of photography/video graph. In recent years, the use of drones by civilians increased rapidly which provides many different applications for citizen such as transportation of important and emergency things. These different applications require for the drones to be tracked during the mission of transportation until reaching to the specified location and return to home [1]. The tracking system for the multiple drones need to GPS drone navigation system for detecting the estimated position of the drones and the tracking algorithm for processing the received information and keeping track. More advanced drones make use of GPS receivers for the navigation and control loop. The global positioning system (GPS) receiver is a satellite navigation system that uses a radio receiver to collect signals from orbiting satellites to determine position, speed, and time [2,3]. This navigation system is more accurate than over forms of navigation, and provides position knowledge with error of a few meters. Advanced GPS systems can provide even better accuracies in error within a few centimeters. The collected information from GPS receiver broadcast its information automatically that will be received by ground control station (GCS) upon the link of network wireless communication [4]. The GPS ground station gather the information from all flying drones which will be processed by the CPU attached to this station as shown in Fig.1. Some issues may be occur during the receiving of GPS information from multiple moving drones such as losing information at instant time and receiving information with error or receiving information with unexpected position. To overcome these issues multiple target tracking with different techniques based on kalman filter or particle filter, and so on will be used [5]. The accuracy of estimated position during tracking depends on the choosing technique. In this situation the algorithm of Markov Chain Monte Carlo -based particle filter (MCMC-PF) will be used for tracking the estimated position instead of the algorithm of Kalman filter [6]. Using MCMC-PF improves the accuracy and overcome the issue of losing information, maneuvering target for the tracked moving drones. Due to the missed spaced and highly maneuvering target, the mainly considered problem for estimation, traditionally, is tried to be solved using linearized filters, such as the Kalman filter (KF) [5].

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When dealing with non-linear models in state equation and measurement relation and a non-Gaussian noise assumption, these estimation methods may lead to non-optimal solutions. The sequential Monte Carlo methods, or particle filters [7,8], provide general solutions to many problems where linearization and Gaussian approximations are difficult. Recently MCMC-based particle filter (MCMC-PF) [9-11] has captured the attention of many researchers in various applications that deal with poor information, difficult nonlinear and/or non Gaussian problems for tracking missed spaced and maneuvering target drones. In MCMC-PF, the particles are sampled from the target posterior distribution via direct MCMC sampling method, which avoids sample impoverishment. To increase the robustness of the algorithm and tracking missed spaced with highly maneuvering drone, the proposed algorithm MCMC-PF can be introduced. Simulation results showed better performance and more effective at tracking when compared to the conventional KF algorithm.

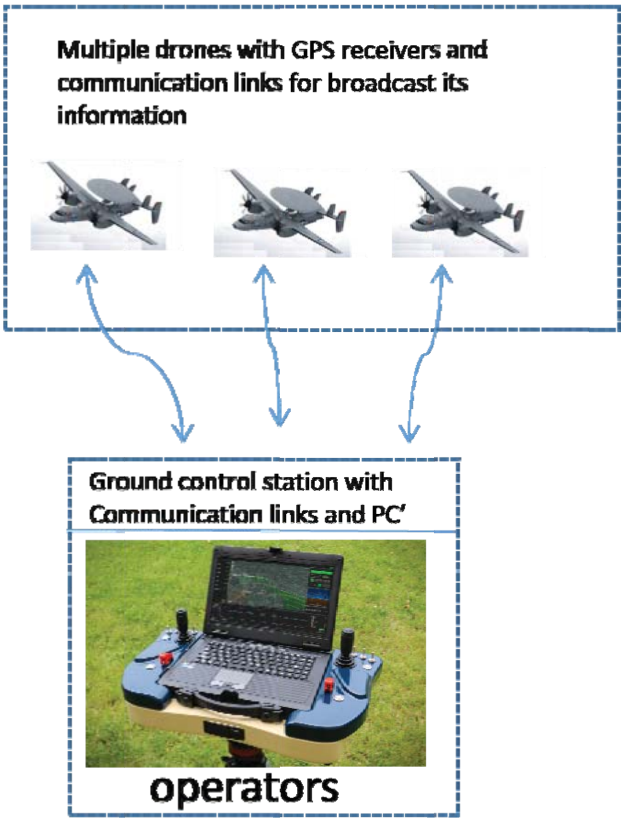

Fig.1 Ground control station for receiving information and tracking multiple drones via wireless communication link

II. PROBLEM FORMULATION

Consider that there are T targets being tracked at time index k. With time evolution, the state $x_t(k) \in R^n$ of a discrete-time, dynamic systems is described by the following equation

$$x_t(k) = f(x_t(k-1), w_t(k-1)) \qquad t = 1,2,\dots,T \tag{1}$$

where f is the system transition function and $w_t(k-1) \in R^n$ is a dynamic noise which has a known probability density function (PDF). The superscript t corresponds to the t^{th} target. The initial target state, $x_t(0)$ for t = 1,2, ..., T , is assumed to be known. At discrete times, the measurements $z(k) \in R^m$ of the state $x_t(k)$ become available and are related to state through the observation equation

$$z(k) = h(x_t(k), v(k)) \qquad t = 1,2 \ldots T \tag{2}$$

where h is a nonlinear measurement function and $v(k) \in R^m$ is a sequence of observation noises of known PDF. From the Bayesian perspective, the tracking problem is to recursively calculate the posterior probability density function, $p(x_t(k) \mid z_{1:k})$,where $z_{1:k} = \{z_m,\ m = 1, \ldots\ldots k\}$.

A. Kalman Filter Theory

Based on Kalman filter estimation [12], we list the filter model. The dynamic state and measurement model of target *t* can be represented as follows

$$x^t(k) = A^t(k-1)x^t(k-1) + w^t(k-1) \qquad t = 1,2,\ldots,T \tag{3}$$

$$z^t(k) = H^t(k)x^t(k) + v^t(k) \qquad t = 1,2 \ldots T \tag{4}$$

Where $x^t(k-1)$ is the n x 1 target state vector. This state can include the position and velocity of the target in space $x = (x, y, \dot{x}, \dot{y})'$, The initial target state, $x^t(0)$ for $t = 1, 2, \ldots, T$, is assumed to be Gaussian With mean m_0^t and known covariance matrix p_0^t. Where the unobserved signal (hidden states) $\{x^t(k)\}$ be modeled as a Markov process of transition probability $p\left(x^t(k) \mid x^t(k-1)\right)$ and initial distribution $p\left(x^t(0)\right) = N(x^t(0); m_0^t, p_0^t)$. $z^t(k)$ is the m **x** 1 measurement vector, $A^t(k-1)$ denotes state transition matrix, $H^t(k)$ denotes measurement matrix, $w^t(k-1)$ and $v^t(k)$ are mutually independent white Gaussian noise with zero mean, and with covariance matrix $Q(k-1)$ and $R(k)$, respectively.

The innovation mean (residual error) of measurement $z_i(k)$ is given by

$$e_i^t(k) = z_i(k) - \hat{z}^t(k) \tag{5}$$

where

$$\hat{z}^t(k) = H^t(k)\overline{m}^t(k) \tag{6}$$

and the predicted state mean and covariance is defined as

$$\overline{m}^t(k) = A^t(k)m^t(k-1) \text{ and } \overline{p}^t(k) = A^t(k)p^t(k-1)A^t(k)' + Q \tag{7}$$

Then, we can update state by

$$m^t(k) = \overline{m}^t(k) + K^t(k)e_{sel}(k) \tag{8}$$

where $e_{sel}(k)$ is the selected innovation mean from $e_i^t(k)$ corresponding to the choosing measurement as a result of data association process , $K^t(k)$ denotes gain matrix calculated by state covariance $p^{t}(k)$ and innovation covariance $S^{t}(K)$, their recursive equations can be represented as follows

$$p^{t}(k)=\overline{p}^{t}(k)-K^{t}(k)S^{t}(K)K^{t}(k)' \tag{9}$$

$$S^{t}(K)=H^{t}(k)\overline{p}^{t}(k)H^{t}(k)'+R(K) \tag{10}$$

$$K^{t}(k)=\overline{p}^{t}(k)-H^{t}(K)S^{t}(K)^{-1} \tag{11}$$

When multiple target (drone) tracking begins, we get for each target *t* measurements within correlation gate (gate size) as candidate measurements when $z_i(k)$ satisfies condition

$$\left(z_i(k)-H^{t}(k)\overline{m}^{t}(k)\right)' S^{t}(k)^{-1}\left(z_i(k)-H^{t}(k)\overline{m}^{t}(k)\right)\leq\gamma \tag{12}$$

where γ denotes correlation gate. If there is only one measurement, this can be used for track update directly; otherwise if there is more than one measurement due to closed spaced target, we need to calculate the equivalent measurement that related to small residual error $e_{sel}(k)$ as described in algorithm 1.

Algorithm 1: Find the validate measurement using the gate of KF algorithm

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1. Find validated region for measurements at time *k*:

$$\overline{Z}^{t}(k)=\{z_j(k)\}, \qquad j=1,\ldots mj$$

By accepting only those measurements that lie inside the gate t

$$\overline{Z}^{t}(k)=\left\{\ Z\ :\left(z_j(k)-\hat{z}^{t}(k)\right)' S^{t}(k)^{-1}\left(z_j(k)-\hat{z}^{t}(k)\right)\leq\gamma\ \right\}$$

Where $e_j(k)=z_j(k)-\hat{z}^{t}(k)$

2. Find the valid measurement j^{*} that has the minimum distance corresponding to $e_{sel}(k)$

3. The associated measurement $z(k)$ is set to

$$z(k)=z_{j^{*}}(k)$$

B. Markov Chain Monte Carlo based Particle Filter (MCMC -PF)

1) Basic theory of particle filter

The particle filter deals with complicated nonlinear and /or non-Gaussian problems [8,13-17]. The basic idea of this method is Monte Carlo simulation, in which the posterior density is approximated by a set of particles with associated weights $\left\{x_t^{i}(k-1),w_t^{i}(k-1)\quad i=1,2,\ldots N\right\}$. At every time step we sample from the proposal distribution $q(x_t(k)|x_t(k-1),z_{1:k})$ to achieve new particles and compute new weights according to the particle likelihoods. After normalization of weights, the posterior density can be represented by $\left\{ (x_t^i(k), w_t^i(k) \quad i=1,2,...N \right\}$. The implementation of the basic particle filter is as follows:

(1) Initialization

Draw a set of particles from the prior $p(x_t(0))$:

$$x_t^i(0) \sim p(x_t(0)), \quad i=1,2...,N \tag{13}$$

Prediction:

(2) Sampling step

(a) for i=1,2,…N

Sample $x_t^i(k)$ from the proposal distribution $q(x_t(k)\,|\,x_t(k-1), z_{1:k})$:

$$x_t^i(k) \sim q(x_t(k)\,|\,x_t(k-1), z_{1:k}) \tag{14}$$

Update:

(b) Evaluate the importance weights when the candidate measurement $z(k)$ is choosing by data association algorithm as in algorithm 1

$$w_t^i(k) = w_t^i(k-1)\frac{p(z(k)\,|\,x_t^i(k))p(x_t^i(k)\,|\,x_t^i(k-1)}{q(x_t(k)\,|\,x_t(k-1), z_{1:k})} \tag{15}$$

(c) Normalize the weights

$$\tilde{w}_t^i(k) = w_t^i(k) / \sum_{i=1}^{N} w_t^i(k) \quad , i=1,2...,N \tag{16}$$

(3) Markov Chain Monte Carlo (MCMC) step as described in algorithm 2

(4) Output step

Output a set of particles $\left\{ (x_t^i(k), w_t^i(k) \quad i=1,2,...N \right\}$ that can be used to approximate the posterior distribution.

Expectation:

$$\hat{x}_t(k) = \sum_{i=1}^{N} \tilde{w}_t^i(k) x_t^i(k) \tag{17}$$

Covariance:

$$p_t(k) = \sum_{i=1}^{N} \tilde{w}_t^i(k)(x_t^i(k) - \hat{x}_t(k))(x_t^i(k) - \hat{x}_t(k))' \tag{18}$$

and set $w_t^i(k) = 1/N, \quad i=1,2,...N$ (19)

(5) k=k+1, go to step 2

2) Basic theory of Markov Chain Monte Carlo (MCMC)

The MCMC step, as described in [18] has an invariant distribution $\prod_{i=1}^{N} P(x_{t\,0:k}^i \,|\, z_{1:k})$, which is applied to each of the N particles, one at the time. A Markov Chain is constructed by approximate a candidate for the next state $x_t^{*i}(k)$ given the current state $x_t^i(k)$ according to the proposal distribution $p(x_t(k)|x_t(k-1))$. This state transition is accepted with probability

$$\alpha(x_t^i(k), x_t^{*i}(k)) = min\left\{1, \frac{p(z(k)|x_t^{*i}(k))}{p(z(k)|x_t^i(k))}\right\} \tag{20}$$

Algorithm 2: Markov Chain Monte Carlo (MCMC) Step

(a) For each particle 1:*N* sample the proposal candidate for the approximation of the next state.

$x_t^{*i}(k) \sim p(x_t(k)/x_t(k-1))$

(b) Sample $\rho \sim$ U(0,1),where U(0,1) is a uniform distribution in the interval (0,1).

(c) If $\rho \leq min\left\{1, \frac{p(z(k)|x_t^{*i}(k))}{p(z(k)|x_t^i(k))}\right\}$

then accept move:

$$x_t^i(k) = x_t^{*i}(k)$$

else reject move

$$x_t^i(k) = x_t^i(k)$$

end if.

III. IMPLEMENTATION OF MCMC–PARTICLE FILTER (MCMC-PF)

The algorithm MCMC-PF is an approximate likelihood- based approach for solving the data association problem in multi target tracking when no continuation of the GPS received information or GPS information that related to location of the targets drones is not accurate due to system error. This algorithm depends on the current of detected measurements contained in frame received from GPS receiver at each time interval which is processed by GCS program. At each time interval the received measurements to be processed are contained in a gate. This constructed gate is created for detecting the valid measurement when more than one measurement existed in the current gate. This refer to the one valid measurement in the current gate is assumed to be originated from true target, while the remaining measurements in the gate are considered as invalid that are assumed to be originated from another targets that will be associated to another track. This method is called filtering gate method to select or candidate the one valid measurement for each current track to be processed in the updating step as in algorithm 1. The filtering gate of data association algorithm that is used in KF algorithm can be used in MCMC-PF during implementation to overcome the issues of highly maneuvering or closed spaced multi-targets in the presence of error or absent target location. The proposed algorithm using MCMC-PF is represented in algorithm 3.

Algorithm 3: MCMC-PF

For t=1 to T

Initialization

1- Draw a set of particles from the prior $p(x_t(0)): \; x_t^i(0) \sim p(x_t(0)), \quad i = 1,2...,N$,
$w_t^i(0) = 1/N$

Prediction:

2- With sampling step *Particles at time step k*-1, $\{x_t^i(k-1)\}$, are passed through the dynamic state model as in (1) to obtain the predicted particles
at time step k, $\{x_t^i(k)\}$:

$$x_t^i(k) = f(x_t^i(k-1), \omega_t^i(k-1)) \qquad t = 1,2,...,T\,,\; i=1,2,\ldots N$$

Update:

4- Once the measurement data $z(k)$ is obtained from algorithm 1, evaluate the importance weight of each predicted particle

$$w_t^i(k) = w_t^i(k-1)\, p(z(k) \mid x_i^i(k))$$

and normalize the weights

$$\tilde{w}_t^i(k) = w_t^i(k) / \sum_{i=1}^{N} w_t^i(k) \quad , i = 1,2...,N$$

5- Markov Chain Monte Carlo (MCMC) step as described in algorithm 2

6- Output step

Output a set of particles $\{(x_t^i(k), w_t^i(k) \quad i = 1,2,...N\}$ that can be used to approximate the posterior distribution by Calculating the expectation $\hat{x}_t(k)$ and Covariance $p_t(k)$ using (17),(18) respectively

and set $w_t^i(k)$ as in (19)

7- k=k+1, go to step 2

8- End for

Finally, by using nearest neighbor data association algorithm ,we obtain for each target the valid measurement in the tracked target gate that is applied to MCMC-PF before entering in the process of the updating step . The other measurements in the gate are assumed to be invalid and the updating process for tracking the targets is not assigned to any one of them. MCMC-PF is used to improve the performance in missed spaced for the tracked targets and to maintain the tracking to the targets that move with high maneuvering and not accurate position.

IV. SIMULATION RESULTS

In order to demonstrate the performance of the proposed MCMC-PF algorithm, we provide a comparison with the most popular conventional tracking algorithm KF. One test scenario has been chosen in this section for moving more than one target in the XY(lat-long) plane with different issues according to the two different tracking problems; missed spaced and high maneuvering targets. The discrete time state equation with sampling interval Δt = 4

sec is $x_t(k) = F\, x_t(k-1) + \Gamma\, w_t(k-1) \qquad t = 1,2,..T$

where

$x_t(k) = \left[x(k), y(k), \dot{x}(k), \dot{y}(k) \right]$ is the state vector ; $x(k)$ and $\dot{x}(k)$ are respectively the position and velocity of the moving object along the Cartesian frame X axis; and $y(k), \dot{y}(k)$ along the Y axis. Γ is a unity matrix and $w_t(k-1)$ is discrete time white, Gaussian noise: $w(k) \sim N(0, Q_k)$, $Q = G \;\; G'$. The measurements are received from one GPS drone which is positioned at the origin of the plane. The measurement equation is as follow:

$z(k) = h(x_t(k)) + v(k) \qquad t = 1,2...T$, where $z(k) = [z_1(k), z_2(k)]$ is the observation vector. $z_1(k)$ is related to the latitude and $z_2(k)$ is the longitude standard GPS position which is converted to the Cartesian x,y position. The measurement noise $v(k) = [v_{z1}(k), v_{z2}(k)]$ is a zero mean Gaussian white noise process with variance R , where $R = \begin{bmatrix} \sigma_{z1}^2 & 0 \\ 0 & \sigma_{z2}^2 \end{bmatrix}$,

$$z(k) = \begin{bmatrix} \sqrt{(x(k) - x_r)^2 + (y(k) - y_r)^2} + v_{z1}(k) \\ arctan\left(\dfrac{y(k) - y_r}{x(k) - x_r} \right) + v_{z2}(k) \end{bmatrix}$$

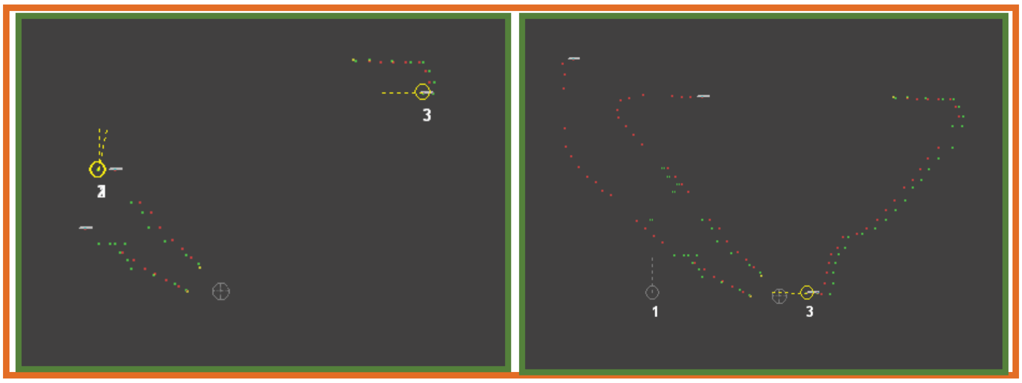
where (x_r, y_r) is the position of the sensor at the origin that is related to the location of ground control station,

$$F = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad G = \begin{bmatrix} \dfrac{\Delta t^2}{2} & 0 \\ 0 & \dfrac{\Delta t^2}{2} \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix}$$

The test scenario has three tracks with initial state $x_1(0)$ = [12.5km, 12.4km, 80m/s, 80m/s], $x_2(0)$ = [12.1km, 12.8km, 80m/s, 80m/s], $x_3(0)$ = [15.8km, 15.8km, 80m/s, 80m/s], which continues from the first frame to the last frame. This scenario is used to evaluate the tracking performance by the MCMC-PF and KF when the targets move with losing their received information at some instant time of frames and with high maneuvering in the presence of inaccurate position. We initiate the other parameters as; the row and column sizes of the volume ($V_s = S_W \times S_H$), V_s=15x20, $T = 38*4 = 152$ sec, in addition to $R = \begin{bmatrix} 400m^2 & 0 \\ 0 & 1deg^2 \end{bmatrix}$, the particle number N=200,sensor position (x_r, y_r)= (0,0)

According to this test scenario, using the KF algorithm to track in environment with missed spaced and with highly maneuvering three targets, failing to track for the two closed spaced targets no#1 and 2 while continuing to track no#3 which its target information has a large distance in position from tracked targets no#1,2 as shown in Fig.3 (a,b,c,d). The GPS mapping position X-Y trajectory is implemented in Fig 4 given a fixed threshold ($\gamma = 10^{-4}$). But using MCMC-PF algorithm, we showed that this algorithm succeeded to track the all the three targets in the same environment as shown in Fig. 5(a,b). We obtain GPS mapping position trajectories for X- and Y- components as shown in Fig. 6. In these figures, the colored solid with dotted line represents the underlying truth targets of the trajectory (each target with different color) while the colored + symbol represents trajectory of the estimated tracked targets. The MCMC-PF algorithm (+ symbol with different color) detects and associates the proper sequence of observation very well compared to KF which fails to continue for some tracks(no#1,2). According to the test scenario we also compared error root mean square value (RMSE) for the different two approaches with three targets as shown in Fig. 7. MCMC-PF algorithm has lower error, RMSE values and less sensitive to lose information and inaccurate position than KF over frame numbers.

(a) (b)



(c) (d)

Fig. 3. The state of tracking multi-moving drones (3 moving targets) using the approache of KF (from (a)-(d)) failing to track for two closed spaced targets no#1,no#2 due to losing in received GPS information at some instant time. While at the same environment as in (d) KF succeeded to continue tracking for track no#3 that is consederd in processing as a single target due to moving in far distance from target no#1,no#2 .

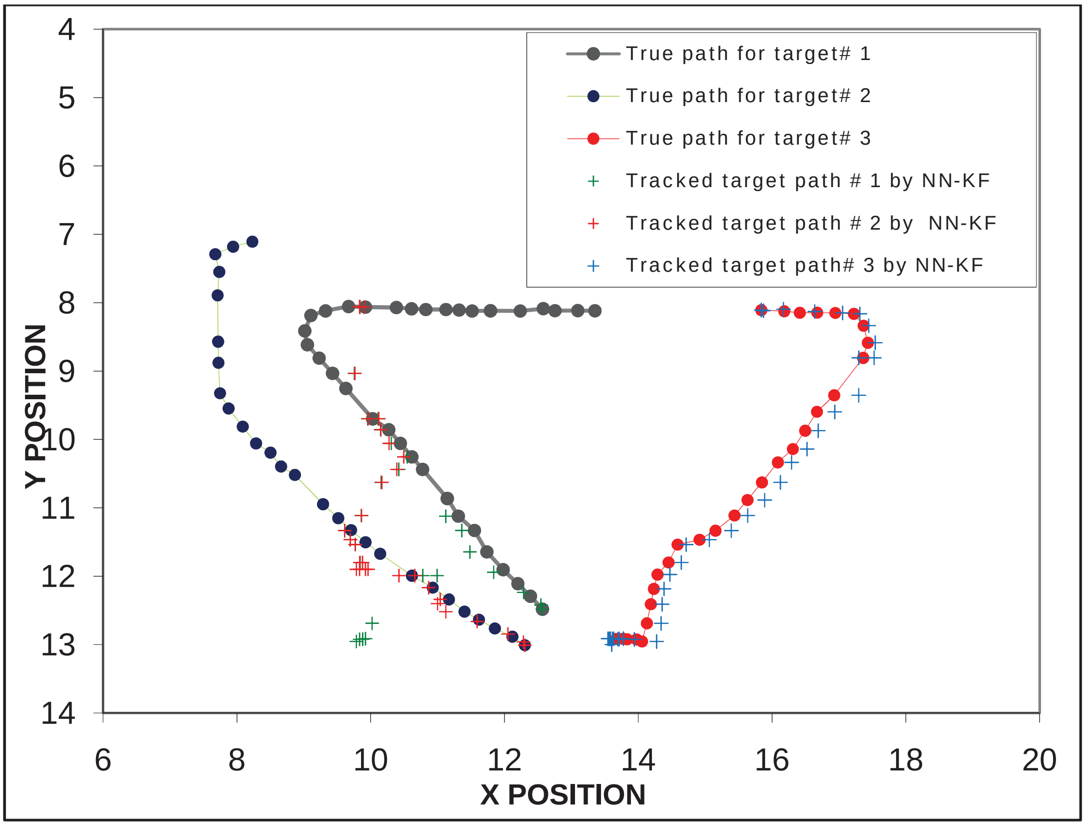
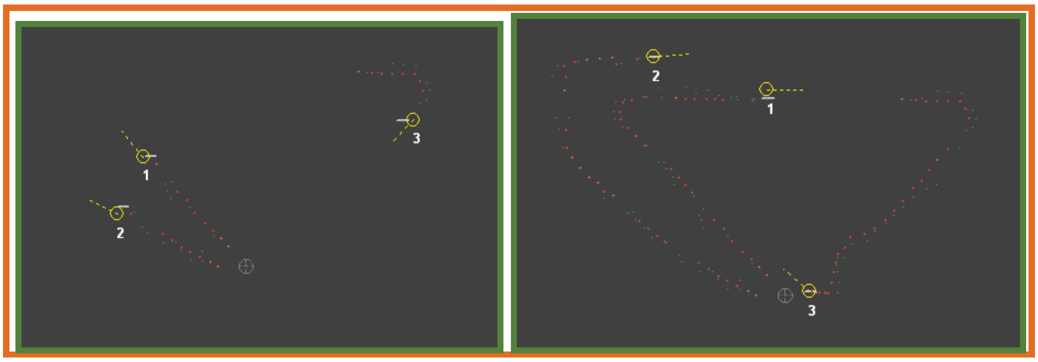


Fig. 4. Maping of Trajectory for X-Y components using the KF to track 3 targets (+ symbol) with losing in information from the GPSreceivers all showed the true targets path as in solid with dotted line.. this showed failing to tracks no#1,2



(a) (b)

Fig. 5. The state of tracking multi-moving drones (3 moving targets) using the approach of MCMC-PF (from (a)-(b)) succeeded to continue tracking for all the targets in spite of losing in received GPS information at some instant time and with high maneuvering.

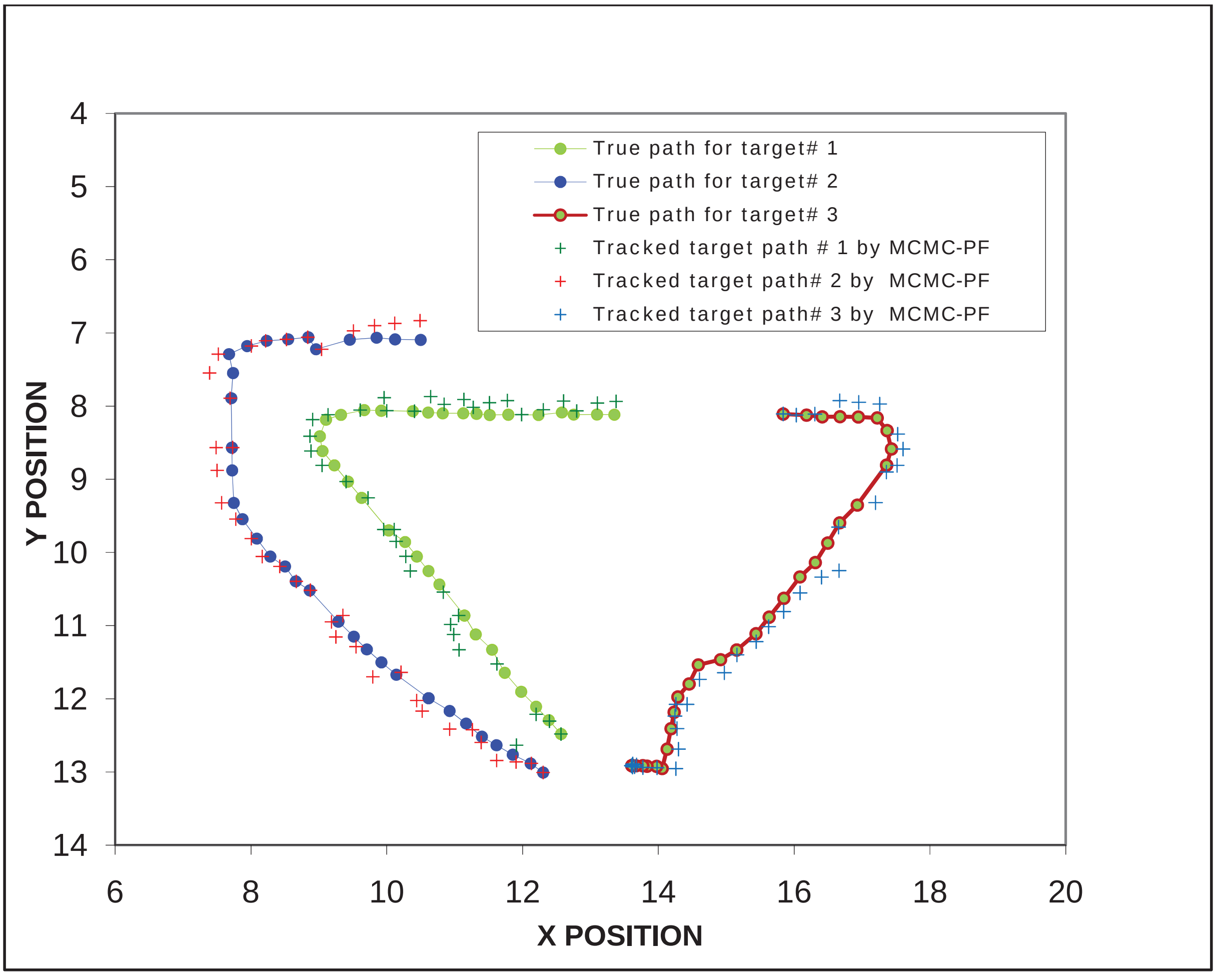


Fig. 6. Maping of Trajectory for X-Y components using the MCMC-PF to track 3 targets (+ symbol) with losing in information from the GPSreceivers all showed the true targets path as in solid with dotted line. this showed succeding to track all the targets drones.

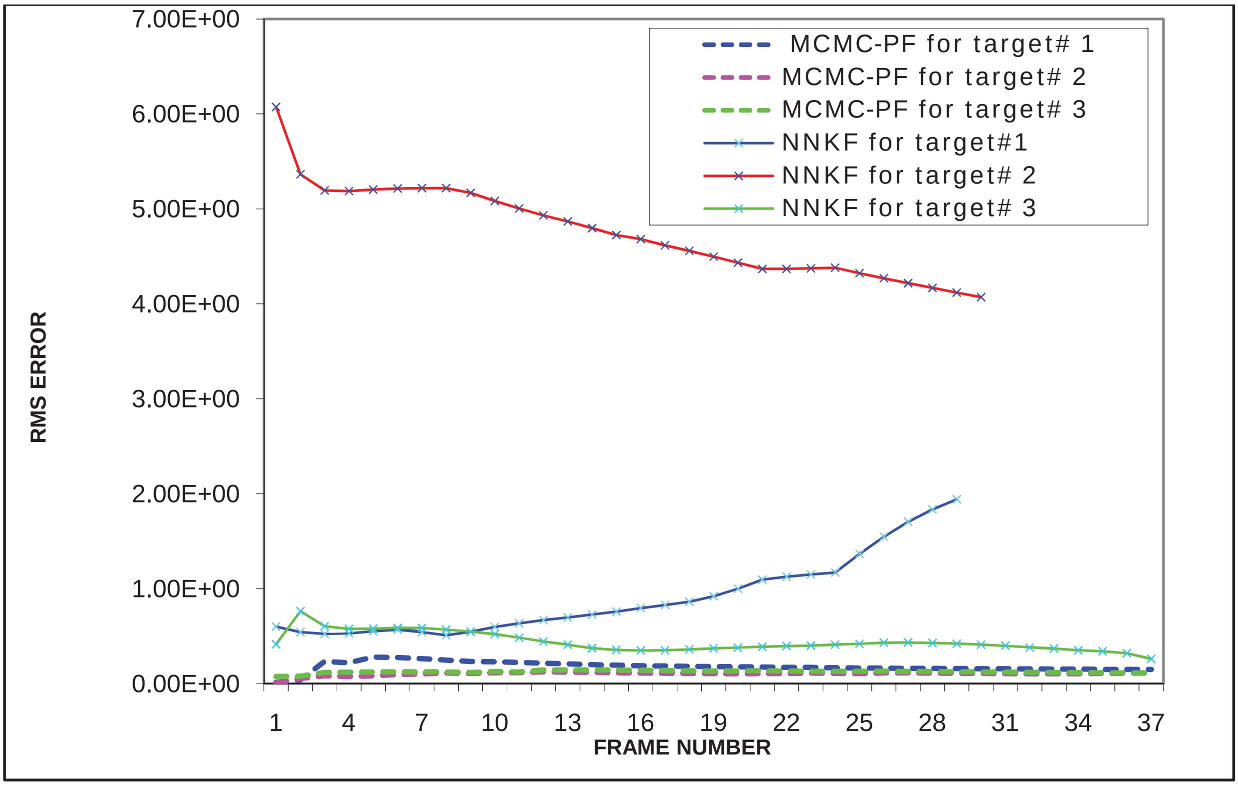


Fig.7. The RMSE for each target (3 targets) separately over frame number according to tracking evaluation of our scenario as shown in Fig. 4,6 using the two approaches KF,MCMC-PF.

V. Conclusion

In this paper for multi-target tracking using GPS information, the MCMC-PF algorithm can solve the tracking problem effectively, such as; missed spaced, highly maneuvering target with inaccurate position when compared to KF algorithm. From the results obtained in the simulations, we have showed that the KF fails to track the targets that have losing in information, inaccurate position from time to time especially for multiple targets moving in closed spaced and with high maneuvering. Where the MCMC-PF algorithm is capable of tracking the targets and avoids the issues of losing received GPS information or received information with error in position. Thus, the last algorithm during existing of these issues improves the data association process which has been shown to give drones targets the ability to continue to be tracked and also improves the accuracy of tracking using GPS receiver.

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